Communication Systems

Lecture 2

Fourier Series in Communication

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Chapter 2

The Fundamentals of Electronics: A Review

Topics Covered in Chapter 2

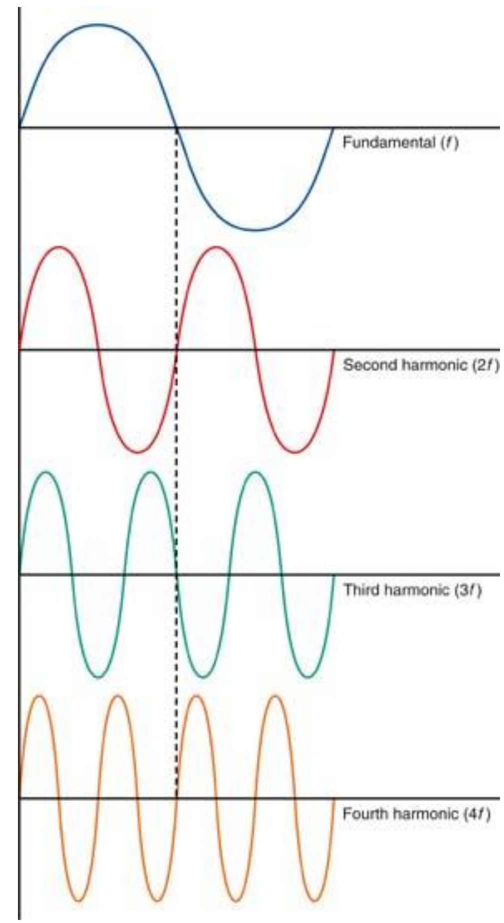
- 2-1: Gain, Attenuation, and Decibels
- 2-2: Tuned Circuits
- 2-3: Filters
- 2-4: Fourier Theory

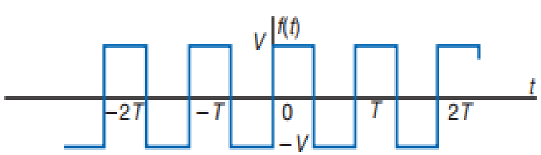
2-4: Fourier Theory

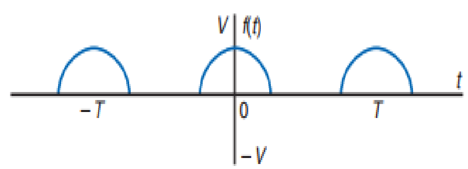
- The mathematical analysis of modulation and multiplexing methods in communication systems assumes sine wave carriers and information signals.
- In the real world, not all information signals are sinusoidal. Information signals are typically more complex
- One method used to determine the characteristics and performance of a communication circuit or system, specifically for non-sine wave approach, is **Fourier analysis.**

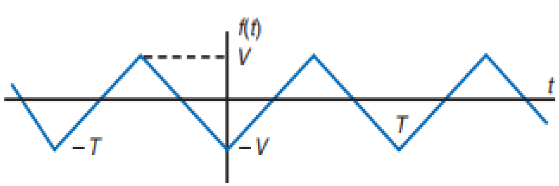
2-4: Fourier Theory

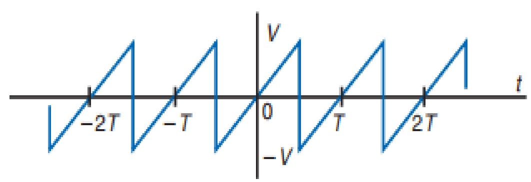
- The Fourier theory states that a non-sinusoidal waveform can be broken down into individual harmonically related sine wave or cosine wave components.
- A **square, Triangle, half-wave and full-wave rectified signals are** classic examples of this phenomenon.







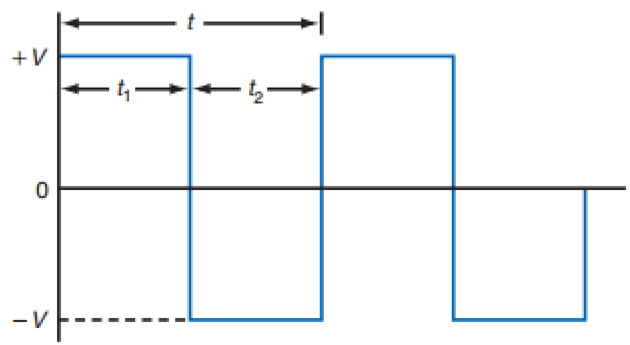


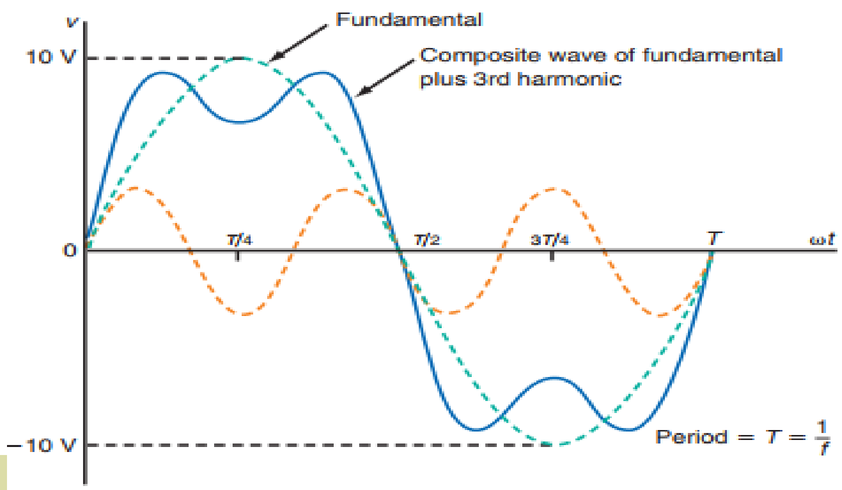


2-4: Fourier Theory

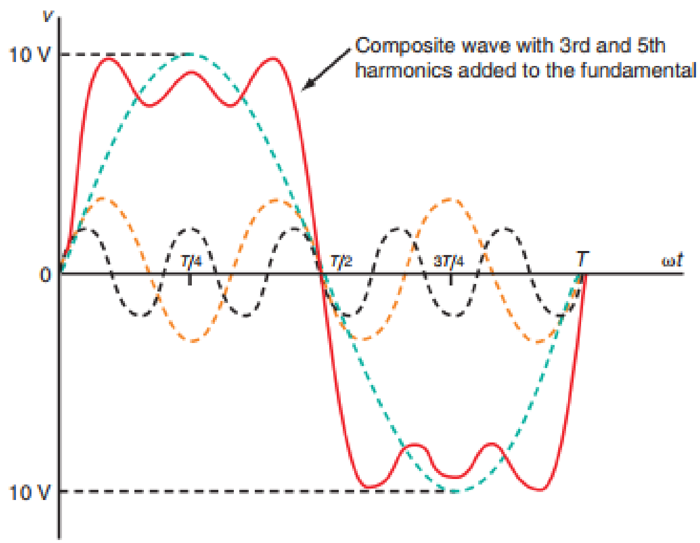
- Fourier analysis states that a non-sinusoidal wave is made up of the sum of a collection of sine wave at the fundamental frequency of the non-sinusoidal wave plus an infinite number of sin and/or cos harmonics (Fourier mathematical analysis).

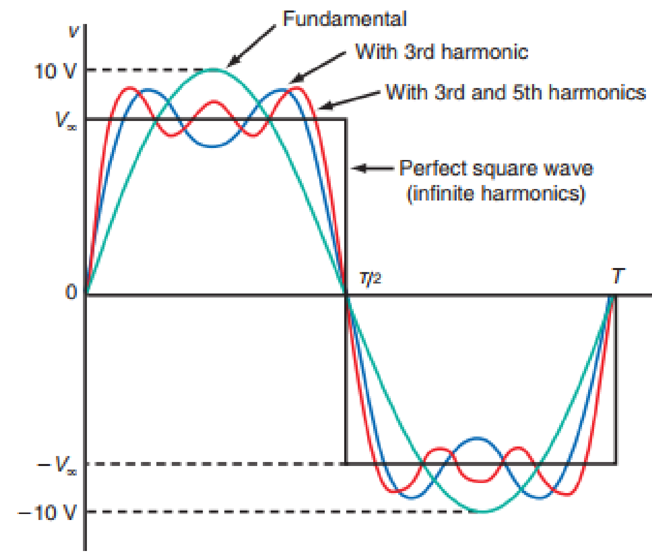
$$f(t) = \frac{4V}{\pi}\left[\sin 2\pi\left(\frac{1}{T}\right)t + \frac{1}{3}\sin 2\pi\left(\frac{3}{T}\right)t + \frac{1}{5}\sin 2\pi\left(\frac{5}{T}\right)t + \frac{1}{7}\sin 2\pi\left(\frac{7}{T}\right)t + \cdots\right]$$



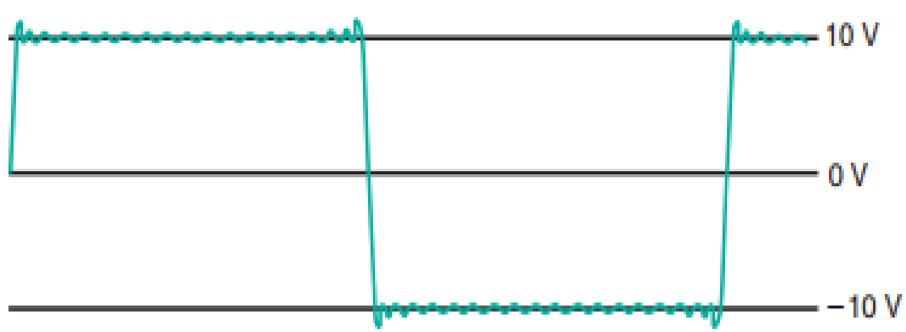


2-4: Fourier Theory





Square wave made up of 20 odd harmonics added to the fundamental.

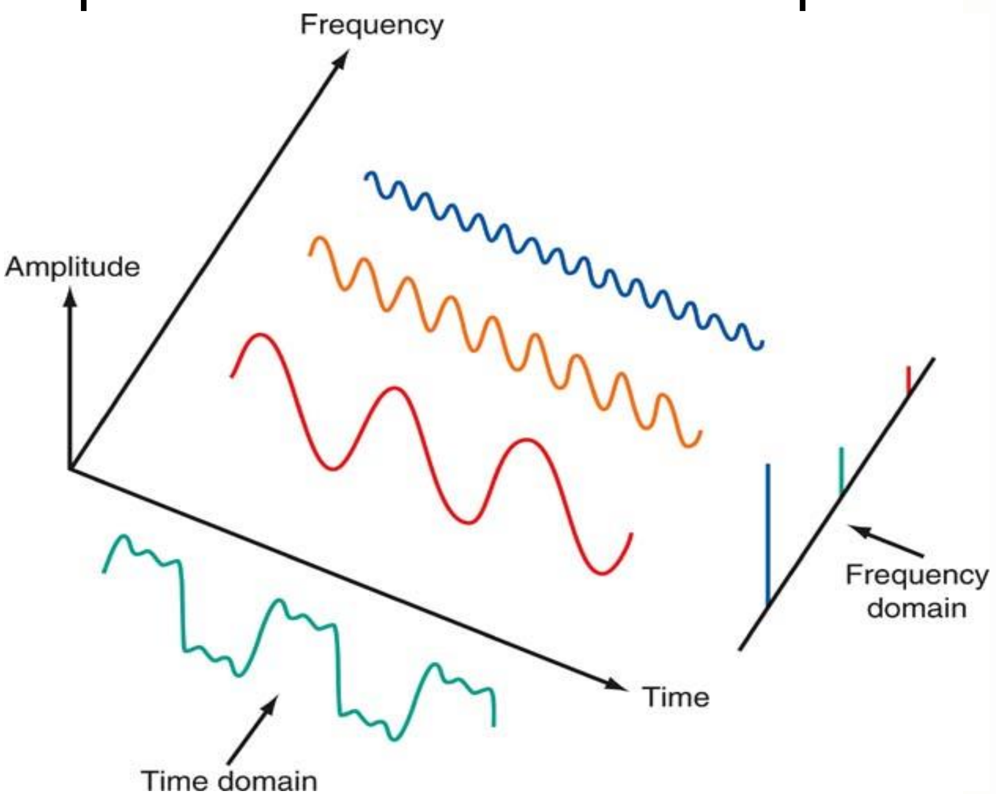


2-4: Fourier Theory

Time Domain Versus Frequency Domain

- Analysis of variations of voltage, current, or power with respect to time are expressed in the **time domain**.
- A **frequency domain** plots amplitude variations with respect to frequency.

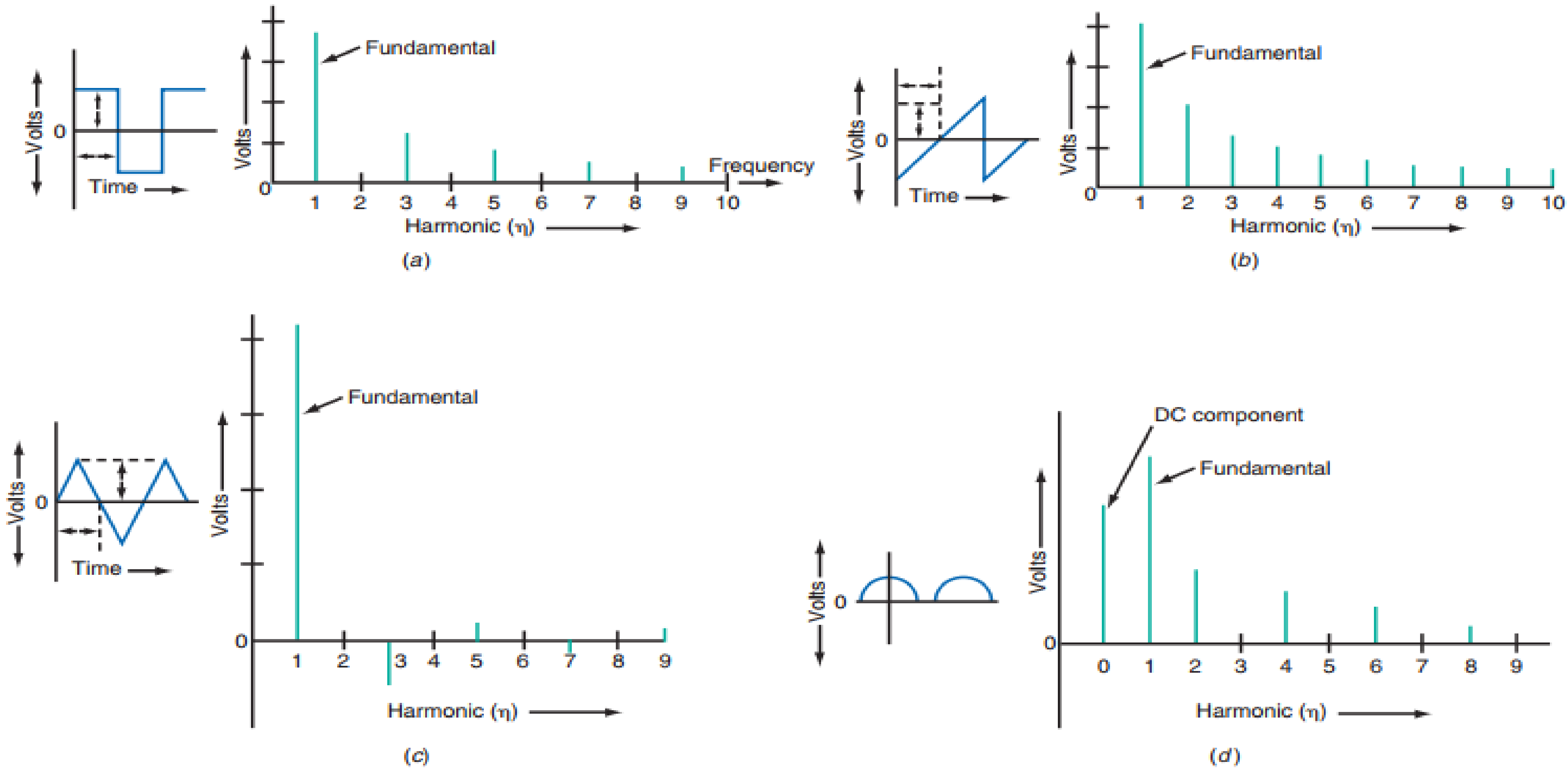
Fourier theory gives us a new way to express complex signals with respect to frequency.



2-4: Fourier Theory

Time Domain Versus Frequency Domain

- A **spectrum analyzer** is an instrument like Oscilloscope used to produce a frequency-domain display.



2-4: Fourier Theory

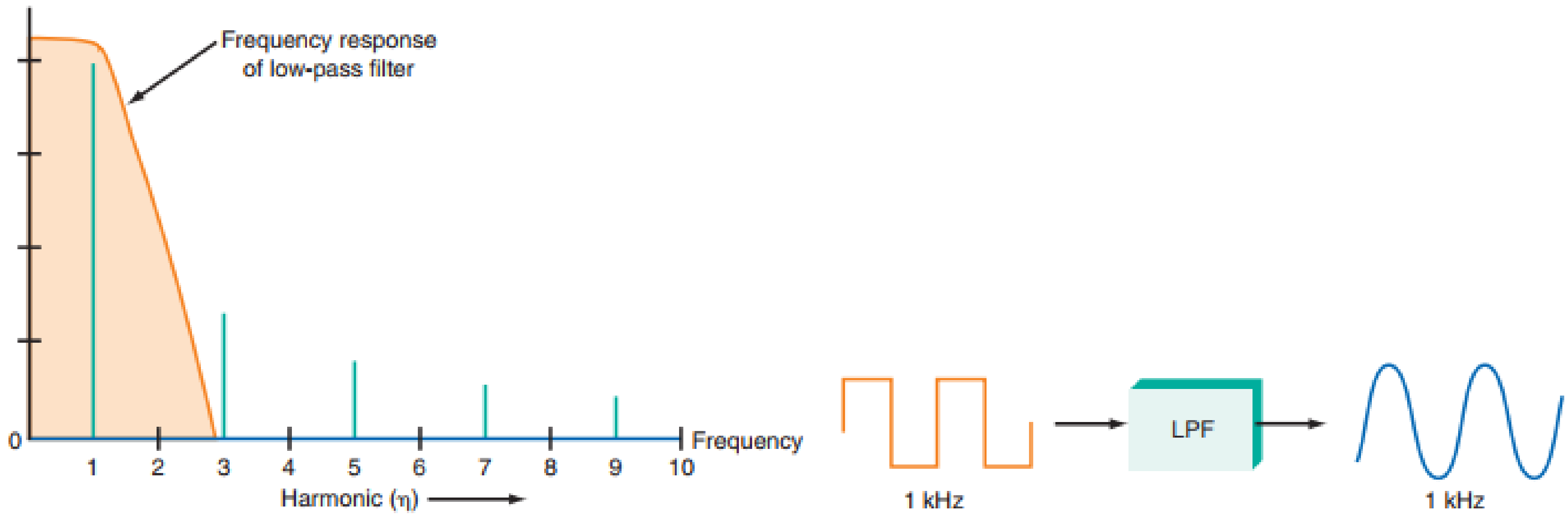
The Importance of Fourier Theory

- Fourier analysis allows us to determine not only sine-wave components in a complex signal but also but also how much bandwidth a particular signal occupies.
- Complex signals obviously take up more spectrum space
- If this signal is to perfectly pass unattenuated and undistorted, then all harmonics must be passed

2-4: Fourier Theory

The Importance of Fourier Theory

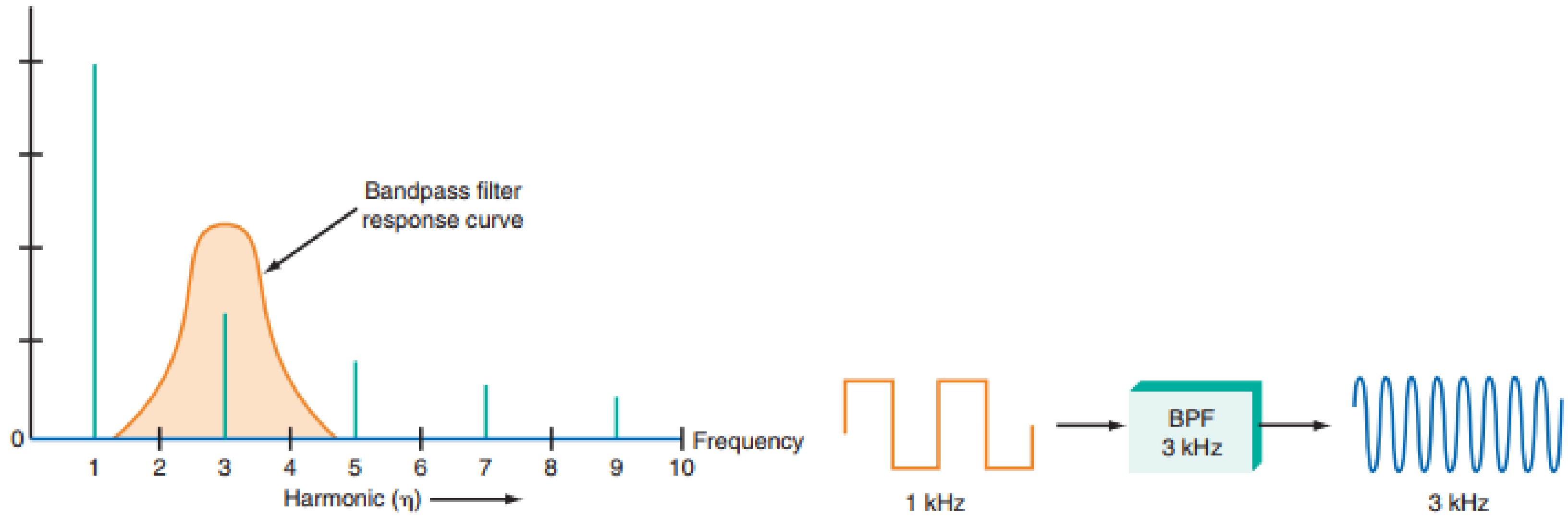
- As an example. If a 1-kHz square wave is passed through a lowpass filter with a cutoff frequency just above 1 kHz, all the harmonics beyond the third harmonic are greatly attenuated or, for the most part, filtered out completely.



2-4: Fourier Theory

The Importance of Fourier Theory

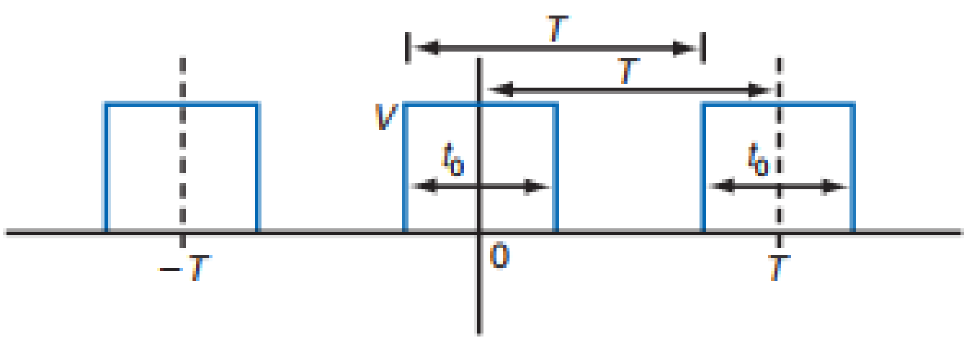
- Another example, If a 1-kHz square wave is passed through a bandpass filter set to the third harmonic, resulting in a 3-kHz sine wave output. In this case, the filter used is sharp enough to select out the desired component.



2-4: Fourier Theory

The Fourier Analysis of the Binary Pulse

- The Fourier analysis of binary pulses is especially useful in communication, as it gives a way to analyze the bandwidth needed to transmit such pulses.
- Theoretically, the system must pass all the harmonics in the pulses, in reality, relatively few must be passed to preserve the shape of the pulse.

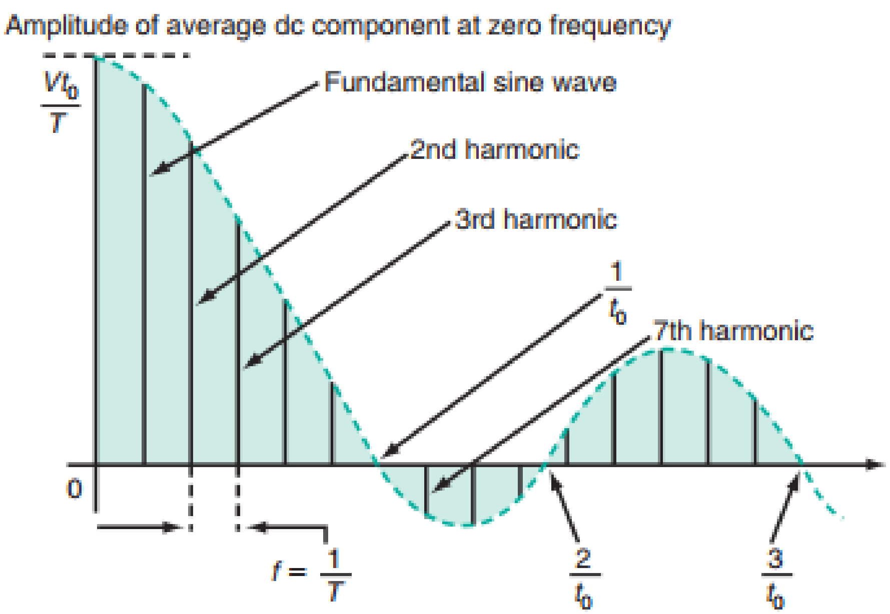


✓ The period is T,
✓ The pulse width is t_0.
✓ The duty cycle is t_0/T.

2-4: Fourier Theory

The Fourier Analysis of the Binary Pulse

- In terms of Fourier analysis, the pulse train is made up of a fundamental and all even and odd harmonics
- A frequency-domain graph of harmonic amplitudes plotted with respect to frequency is shown



- ✓ Each vertical line represents the peak value of the sine wave components
- ✓ Some of the higher harmonics are negative; this means that their phase is reversed.
- ✓ The envelope of the frequency spectrum is a **sinc** function = sin(x)/x

2-4: Fourier Theory

The Fourier Analysis of the Binary Pulse

- The sinc function curve is used in predicting the harmonic content of a pulse train and thus the bandwidth necessary to pass the wave
- For data communication applications, it is generally assumed that a bandwidth equal to the first zero crossing of the envelope is the minimum that is sufficient to pass enough harmonics for reasonable waveshape

$$\mathrm{BW} = \frac{1}{t_0}$$